

Axial–Vector Torsion and the Teleparallel Kerr Spacetime

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In the context of the teleparallel equivalent of general relativity, we obtain the tetrad and the torsion fields of the stationary axisymmetric Kerr spacetime. It is shown that, in the slow rotation and weak field approximations, the axial–vector torsion plays the role of the gravitomagnetic component of the gravitational field, and is thus the responsible for the Lense–Thirring effect.

I. INTRODUCTION

In a metric–affine theory of gravitation [1,2], the metric and the connection are considered as independent variables, and the underlying spacetime presents nonvanishing curvature, torsion and nonmetricity. On the other hand, in the special case of teleparallel gravity [3–6], which is characterized by the vanishing of curvature and nonmetricity, the relevant spacetime is the Weitzenböck spacetime [7]. As is well known, at least in the absence of spinor fields, the teleparallel gravity is equivalent to general relativity. In order to better understand this relationship, a study of the teleparallel version of the exact solutions of general relativity is indispensable [8–14]. For example, in the context of general relativity, the Kerr solution is an axisymmetric *curved* spacetime produced by a spherically symmetric rotating source. In the context of teleparallel gravity, this solution might correspond to an axisymmetric *torsionned* spacetime.

We will use the greek alphabet ($\mu, \nu, \rho, \dots = 1, 2, 3, 4$) to denote tensor indices, that is, indices related to spacetime. The latin alphabet ($a, b, c, \dots = 1, 2, 3, 4$) will be used to denote local Lorentz (or tangent space) indices, whose associated metric tensor is $\eta_{ab} = \text{diag}(+1, -1, -1, -1)$. Tensor and local Lorentz indices can be changed into each other with the use of a tetrad field $h^a{}_\mu$, which satisfies

$$h^a{}_\mu h_a{}^\nu = \delta_\mu{}^\nu; \quad h^a{}_\mu h_b{}^\mu = \delta^a{}_b. \quad (1)$$

A nontrivial tetrad field can be used to define the linear Weitzenböck connection

$$\Gamma^\sigma{}_{\mu\nu} = h_a{}^\sigma \partial_\nu h^a{}_\mu, \quad (2)$$

with respect to which the tetrad is parallel:

$$\nabla_\nu h^a{}_\mu \equiv \partial_\nu h^a{}_\mu - \Gamma^\rho{}_{\mu\nu} h^a{}_\rho = 0. \quad (3)$$

The Weitzenböck connection satisfies the relation

$$\Gamma^\sigma{}_{\mu\nu} = \overset{\circ}{\Gamma}{}^\sigma{}_{\mu\nu} + K^\sigma{}_{\mu\nu}, \quad (4)$$

where

$$\overset{\circ}{\Gamma}{}^\sigma{}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} [\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}] \quad (5)$$

is the Levi–Civita connection of the metric

$$g_{\mu\nu} = \eta_{ab} h^a{}_\mu h^b{}_\nu, \quad (6)$$

and

$$K^\sigma{}_{\mu\nu} = \frac{1}{2} [T_\mu{}^\sigma{}_\nu + T_\nu{}^\sigma{}_\mu - T^\sigma{}_{\mu\nu}] \quad (7)$$

is the contorsion tensor, with

$$T^\sigma{}_{\mu\nu} = \Gamma^\sigma{}_{\nu\mu} - \Gamma^\sigma{}_{\mu\nu} \quad (8)$$

the torsion of the Weitzenböck connection [15].

The torsion tensor can be decomposed into three irreducible parts under the group of global Lorentz transformations [3]. They are the tensor part

$$t_{\lambda\mu\nu} = \frac{1}{2} (T_{\lambda\mu\nu} + T_{\mu\lambda\nu}) + \frac{1}{6} (g_{\nu\lambda} V_\mu + g_{\nu\mu} V_\lambda) - \frac{1}{3} g_{\lambda\mu} V_\nu, \quad (9)$$

the vector part,

$$V_\mu = T^\nu{}_{\nu\mu}, \quad (10)$$

and the axial-vector part

$$A^\mu = \frac{1}{6} \epsilon^{\mu\nu\rho\sigma} T_{\nu\rho\sigma}. \quad (11)$$

In terms of these irreducible components, the torsion tensor is written as

$$T_{\lambda\mu\nu} = \frac{1}{2} (t_{\lambda\mu\nu} - t_{\lambda\nu\mu}) + \frac{1}{3} (g_{\lambda\mu} V_\nu - g_{\lambda\nu} V_\mu) + \epsilon_{\lambda\mu\nu\rho} A^\rho. \quad (12)$$

Relying on the equivalence alluded to above, we are going in this paper to obtain the teleparallel equivalent of the general relativity Kerr solution. This solution will be obtained by solving the zero-curvature field equations with torsion. We will proceed according to the following scheme. In Section II, we obtain the tetrad field, the Weitzenböck connection and the irreducible components of the torsion tensor for the Schwarzschild solution in both isotropic and Schwarzschild coordinate systems. As expected, due to the spherical symmetry, the axial-vector torsion vanishes identically. The weak-field limit is then considered, and we show that the tensor and the vector parts of torsion combine themselves to yield the Newtonian force. In Section III, we obtain the tetrad field, the Weitzenböck connection and the irreducible components of the torsion tensor for the exact Kerr solution. The slow-rotation and weak-field limits are investigated in Section IV, where we show that the axial-vector tensor appears in these limits as the gravitomagnetic component of the gravitational field. Discussions and conclusions are presented in Section V. We use units in which the speed of light is set equal to unit: $c = 1$.

II. THE TELEPARALLEL SCHWARZSCHILD SOLUTION

In the spherical, static and isotropic coordinate system $X^1 = \rho \sin \theta \cos \phi$, $X^2 = \rho \sin \theta \sin \phi$, $X^3 = \rho \cos \theta$, the tetrad components of the Schwarzschild spacetime can be obtained from the line element

$$ds^2 \equiv g_{\mu\nu} dX^\mu dX^\nu = C(\rho) dt^2 - D(\rho) (d\rho^2 + \rho^2 d\Omega^2), \quad (13)$$

where

$$d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2. \quad (14)$$

With the subscript μ denoting the column index, they are given by [3]

$$h^a{}_\mu \equiv \begin{pmatrix} \sqrt{C} & 0 & 0 & 0 \\ 0 & \sqrt{D} & 0 & 0 \\ 0 & 0 & \sqrt{D} & 0 \\ 0 & 0 & 0 & \sqrt{D} \end{pmatrix}, \quad (15)$$

with the inverse

$$h_a{}^\mu \equiv \begin{pmatrix} \sqrt{C^{-1}} & 0 & 0 & 0 \\ 0 & \sqrt{D^{-1}} & 0 & 0 \\ 0 & 0 & \sqrt{D^{-1}} & 0 \\ 0 & 0 & 0 & \sqrt{D^{-1}} \end{pmatrix}. \quad (16)$$

In a isotropic coordinate system, $C(\rho)$ and $D(\rho)$ are given respectively by [16]

$$C(\rho) = \left(1 - \frac{GM}{2\rho}\right)^2 \left(1 + \frac{GM}{2\rho}\right)^{-2} \quad (17)$$

and

$$D(\rho) = \left(1 + \frac{GM}{2\rho}\right)^4, \quad (18)$$

with M the gravitational mass of the central source. The corresponding nonvanishing components of the torsion tensor are $(i, j, k, \dots = 1, 2, 3)$

$$T^0_{0i} = H \frac{\partial \rho}{\partial X^i}; \quad T^i_{ij} = J \frac{\partial \rho}{\partial X^j}, \quad (19)$$

where

$$H = \frac{1}{2} \frac{d}{d\rho} [\ln C(\rho)] = \frac{GM}{2\rho^2} (E + F) \quad (20)$$

$$J = \frac{1}{2} \frac{d}{d\rho} [\ln D(\rho)] = -\frac{GM}{\rho^2} F, \quad (21)$$

with

$$E = \left(1 - \frac{GM}{2\rho}\right)^{-1}; \quad F = \left(1 + \frac{GM}{2\rho}\right)^{-1}. \quad (22)$$

The torsion vector and the axial torsion–vector are, consequently,

$$V_0 = 0; \quad V_i = (H + 2J) \frac{\partial \rho}{\partial X^i} \quad (23)$$

and

$$A^\mu = 0. \quad (24)$$

Now, the Schwarzschild geometry can also be globally represented by the Schwarzschild coordinate system $\{x^\mu\} = (t, r, \theta, \phi)$, with the line element in this case given by

$$ds^2 = g_{00} dt^2 + g_{11} dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2, \quad (25)$$

where

$$g_{00} = (-g_{11})^{-1} = 1 - \frac{r_s}{r}, \quad (26)$$

with $r_s = 2GM$ the Schwarzschild radius. Comparing the line elements in the isotropic and in the Schwarzschild coordinates, given respectively by Eqs. (13) and (25), we see that

$$C(\rho) = g_{00}, \quad \sqrt{D(\rho)} \rho = r, \quad \frac{\partial \rho}{\partial r} = \sqrt{\frac{-g_{11}}{D(\rho)}}. \quad (27)$$

Using the general coordinate transformation

$$h^a_{\mu} = \frac{\partial X^{\nu'}}{\partial X^{\mu}} h^a_{\nu'}, \quad (28)$$

where $\{X^\mu\}$ and $\{X^{\nu'}\}$ are respectively the isotropic and Schwarzschild coordinates, we obtain the tetrad in the Schwarzschild coordinate system:

$$h^a_{\mu} \equiv \begin{pmatrix} \gamma_{00} & 0 & 0 & 0 \\ 0 & \gamma_{11} s \theta c \phi & r c \theta c \phi & -r s \theta s \phi \\ 0 & \gamma_{11} s \theta s \phi & r c \theta s \phi & r s \theta c \phi \\ 0 & \gamma_{11} c \theta & -r s \theta & 0 \end{pmatrix}, \quad (29)$$

where we have introduced the following notations: $\gamma_{00} = \sqrt{g_{00}}$, $\gamma_{ii} = \sqrt{-g_{ii}}$, $s\theta = \sin \theta$, and $c\theta = \cos \theta$. Its inverse is

$$h_a{}^\mu \equiv \begin{pmatrix} \gamma_{00}^{-1} & 0 & 0 & 0 \\ 0 & \gamma_{11}^{-1} s\theta c\phi & r^{-1} c\theta c\phi & -(r s\theta)^{-1} s\phi \\ 0 & \gamma_{11}^{-1} s\theta s\phi & r^{-1} c\theta s\phi & (r s\theta)^{-1} c\phi \\ 0 & \gamma_{11}^{-1} c\theta & -r^{-1} s\theta & 0 \end{pmatrix}. \quad (30)$$

One can easily verify that the relations (1) and (6) between $h^a{}_\mu$ and $h_a{}^\mu$ are satisfied.

From Eqs.(29) and (30), we can now construct the Weitzenböck connection, whose nonvanishing components are:

$$\begin{aligned} \Gamma^0{}_{01} &= [\ln \sqrt{g_{00}}]_{,r} & \Gamma^3{}_{32} &= \Gamma^3{}_{23} = \cot \theta \\ \Gamma^1{}_{11} &= [\ln \sqrt{-g_{11}}]_{,r} & \Gamma^1{}_{22} &= -r/\sqrt{-g_{11}} \\ \Gamma^1{}_{33} &= \Gamma^1{}_{22}(\sin \theta)^2 & \Gamma^2{}_{33} &= -\sin \theta \cos \theta \\ \Gamma^2{}_{21} &= \Gamma^3{}_{31} = 1/r & \Gamma^2{}_{12} &= \Gamma^3{}_{13} = \sqrt{-g_{11}}/r, \end{aligned}$$

where a comma followed by a coordinate denotes a derivative in relation to that coordinate. The corresponding nonvanishing torsion components are:

$$\begin{aligned} T^0{}_{01} &= -[\ln \sqrt{g_{00}}]_{,r} \\ T^2{}_{21} &= -(1 - \sqrt{-g_{11}})/r, \\ T^3{}_{31} &= -(1 - \sqrt{-g_{11}})/r. \end{aligned}$$

Now, as expected, because A^μ represents a deviation from the spherical symmetry [17], the axial-vector torsion vanishes identically for a Schwarzschild spacetime:

$$A^\mu \equiv 0.$$

On the other hand, the vector and the tensor parts of torsion are, respectively,

$$V_1 = - \left[(\ln \sqrt{g_{00}})_{,r} + \frac{2(1 - \sqrt{-g_{11}})}{r} \right], \quad (31)$$

and

$$t_{001} = -\frac{1}{3}(g_{00})_{,r} + \frac{2g_{00}}{3r}(1 - \sqrt{-g_{11}}) \quad (32)$$

$$t_{122} = -\frac{r}{2}(1 - \sqrt{-g_{11}}) + \frac{r^2}{6} \left(\frac{(g_{00})_{,r}}{2g_{00}} + \frac{2(1 - \sqrt{-g_{11}})}{r} \right) \quad (33)$$

$$t_{331} = -2 \sin^2 \theta \left[-\frac{r(1 - \sqrt{-g_{11}})}{2} + \frac{r^2}{6} \left(\frac{(g_{00})_{,r}}{2g_{00}} + \frac{2(1 - \sqrt{-g_{11}})}{r} \right) \right]. \quad (34)$$

In the teleparallel description of gravitation, torsion plays the role of the gravitational force. In fact, a spinless particle submitted to a gravitational field will obey the force equation [4]

$$\frac{du_\rho}{ds} - \Gamma_{\mu\rho\nu} u^\mu u^\nu = T_{\mu\rho\nu} u^\mu u^\nu. \quad (35)$$

It is then an easy task to verify that, in the weak-field limit, the vector and the tensor parts of the Schwarzschild torsion combine themselves to yield the Newtonian force,

$$m \frac{d\mathbf{u}}{dt} = -\frac{GMm}{r^2} \hat{\mathbf{r}}, \quad (36)$$

where $\mathbf{u} = (u_r, u_\theta, u_\phi)$, with $\hat{\mathbf{r}}$ the unit vector in the radial direction.

III. THE TELEPARALLEL KERR SOLUTION

The gravitational field of a rotating mass is described by the axially symmetric stationary Kerr metric [16],

$$ds^2 = g_{00}dt^2 + g_{11}dr^2 + g_{22}d\theta^2 + g_{33}d\phi^2 + 2g_{03}d\phi dt, \quad (37)$$

where

$$g_{00} = 1 - \frac{r_s r}{\rho^2}; \quad g_{11} = -\frac{\rho^2}{\Delta}; \quad g_{22} = -\rho^2 \quad (38)$$

$$g_{33} = -\left(r^2 + a^2 + \frac{r_s r a^2}{\rho^2} \sin^2 \theta\right) \sin^2 \theta \quad (39)$$

$$g_{03} = g_{30} = \frac{r_s r a}{\rho^2} \sin^2 \theta \quad (40)$$

with

$$\Delta = r^2 - r_s r + a^2 \quad \text{and} \quad \rho^2 = r^2 + a^2 \cos^2 \theta. \quad (41)$$

In these expressions, a is the angular momentum of a gravitational unit mass source. If $a = 0$, the Kerr metric becomes the Schwarzschild metric in the standard form.

The corresponding Kerr tetrad is

$$h^a{}_\mu \equiv \begin{pmatrix} \gamma_{00} & 0 & 0 & \eta \\ 0 & \gamma_{11} s \theta c \phi & \gamma_{22} c \theta c \phi & -k s \phi \\ 0 & \gamma_{11} s \theta s \phi & \gamma_{22} c \theta s \phi & k c \phi \\ 0 & \gamma_{11} c \theta & -\gamma_{22} s \theta & 0 \end{pmatrix}, \quad (42)$$

with its inverse given by

$$h_a{}^\mu \equiv \begin{pmatrix} \gamma_{00}^{-1} & 0 & 0 & 0 \\ -k g^{03} s \phi & \gamma_{11}^{-1} s \theta c \phi & \gamma_{22}^{-1} c \theta c \phi & -k^{-1} s \phi \\ k g^{03} c \phi & \gamma_{11}^{-1} s \theta s \phi & \gamma_{22}^{-1} c \theta s \phi & k^{-1} c \phi \\ 0 & \gamma_{11}^{-1} c \theta & -\gamma_{22}^{-1} s \theta & 0 \end{pmatrix}, \quad (43)$$

where

$$k^2 = \eta^2 - g_{33} \quad \text{and} \quad \eta = \frac{g_{03}}{\gamma_{00}}. \quad (44)$$

One can verify that the relations (1) and (6) between $h^a{}_\mu$ and $h_a{}^\mu$ are in fact satisfied by the Kerr tetrad. Moreover, analogously to the Kerr metric, the Kerr tetrad reduces to the Schwarzschild tetrad for $a = 0$.

The nonvanishing components of the Weitzenböck connection are:

$$\begin{aligned} \Gamma^0{}_{01} &= [\ln \sqrt{g_{00}}]_{,r} & \Gamma^0{}_{13} &= k g^{03} \gamma_{11} s \theta \\ \Gamma^0{}_{23} &= -k g^{03} \gamma_{22} c \theta & \Gamma^0{}_{31} &= \eta_{,r} / \gamma_{00} - (k^2)_{,r} g^{03} / 2 \\ \Gamma^0{}_{32} &= \eta_{,\theta} / \gamma_{00} - (k^2)_{,\theta} g^{03} / 2 & & \\ \Gamma^1{}_{11} &= [\ln \sqrt{-g_{11}}]_{,r} & \Gamma^1{}_{12} &= [\ln \sqrt{-g_{11}}]_{,\theta} \\ \Gamma^1{}_{22} &= -\gamma_{22} / \gamma_{11} & \Gamma^1{}_{33} &= -k s \theta / \gamma_{11} \\ \Gamma^2{}_{12} &= \gamma_{11} / \gamma_{22} & \Gamma^2{}_{21} &= [\ln \sqrt{-g_{22}}]_{,r} \\ \Gamma^2{}_{22} &= [\ln \sqrt{-g_{22}}]_{,\theta} & \Gamma^2{}_{33} &= -k c \theta / \gamma_{22} \\ \Gamma^3{}_{13} &= \gamma_{11} s \theta / k & \Gamma^3{}_{23} &= \gamma_{22} c \theta / k \\ \Gamma^3{}_{31} &= [\ln k]_{,r} & \Gamma^3{}_{32} &= [\ln k]_{,\theta}. \end{aligned}$$

The corresponding non-zero torsion components are:

$$\begin{aligned} T^0{}_{01} &= -[\ln \sqrt{g_{00}}]_{,r} \\ T^0{}_{13} &= \eta_{,r} / \gamma_{00} - k g^{03} (k_{,r} - \gamma_{11} s \theta) \\ T^0{}_{23} &= \eta_{,\theta} / \gamma_{00} - k g^{03} (k_{,\theta} - \gamma_{22} c \theta) \\ T^1{}_{12} &= -[\ln \sqrt{-g_{11}}]_{,\theta} \\ T^2{}_{12} &= [\ln \sqrt{-g_{22}}]_{,r} - \gamma_{11} / \gamma_{22} \\ T^3{}_{13} &= (k_{,r} - \gamma_{11} s \theta) / k \\ T^3{}_{23} &= (k_{,\theta} - \gamma_{22} c \theta) / k. \end{aligned}$$

The non-zero components of the vector torsion are, consequently,

$$V_1 = -[\ln \sqrt{g_{00}}]_{,r} - [\ln \sqrt{-g_{22}}]_{,r} + \gamma_{11}/\gamma_{22} - [\ln k]_{,r} + \gamma_{11}s\theta/k \quad (45)$$

$$V_2 = -[\ln \sqrt{-g_{11}}]_{,\theta} - [\ln k]_{,\theta} + \gamma_{22}c\theta/k, \quad (46)$$

whereas the torsion axial-vector components are

$$A^{(1)} \times (6h) = -2(g_{00}T^0_{23} + g_{03}T^3_{23}) \quad (47)$$

$$A^{(2)} \times (6h) = 2[g_{00}T^0_{13} + g_{03}(T^3_{13} + T^0_{01})], \quad (48)$$

where we have made the identification

$$h = \sqrt{-g}, \quad (49)$$

with $h = \det(h^a_{\mu})$ and $g = \det(g_{\mu\nu})$.

IV. SLOW-ROTATION AND WEAK-FIELD APPROXIMATIONS

In the case of slow rotation, we keep the terms up to first order in the angular momentum a . The related quantities are simplified as follows:

$$\Delta = r^2 - r_s r; \quad \rho^2 = r^2 \quad (50)$$

$$g_{00} = (-g_{11})^{-1} = 1 - \frac{r_s}{r}; \quad g_{22} = -r^2 \quad (51)$$

$$g_{33} = -r^2 \sin^2 \theta; \quad g_{03} = \frac{r_s a}{r} \sin^2 \theta. \quad (52)$$

In this approximation, both the vector and the tensor parts of torsion reduce to the values of the Schwarzschild solution. On the other hand, in the weak-field limit, characterized by keeping terms up to first order in α , the nonzero components of the axial-vector torsion become

$$A^{(1)} \times (6h) = -2(\gamma_{00} \eta)_{,\theta} = -2(g_{03})_{,\theta} \quad (53)$$

$$A^{(2)} \times (6h) = 2[\gamma_{00}(\eta)_{,r} - \eta(\gamma_{00})_{,r}], \quad (54)$$

where now

$$h = r^2 \sin \theta. \quad (55)$$

Substituting Eqs.(51) and (52), we obtain

$$A^{(1)} = -\frac{2}{3} \frac{\alpha a}{r^2} \cos \theta \quad (56)$$

and

$$A^{(2)} = -\frac{\alpha a}{3r^3} \sin \theta, \quad (57)$$

where $\alpha = (r_s/r)$. In spacelike vector form, the axial-vector becomes,

$$\mathbf{A} = A^{(1)}\gamma_{11} \mathbf{e}_r + A^{(2)}\gamma_{22} \mathbf{e}_\theta, \quad (58)$$

that is,

$$\mathbf{A} = \frac{\alpha a}{3r^2} [2 \cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta]. \quad (59)$$

It has been shown by many authors [3,17–23] that the spin precession of a Dirac particle in torsion gravity is intimately related to the axial-vector,

$$\frac{d\mathbf{s}}{dt} = -\mathbf{b} \times \mathbf{s} \quad (60)$$

where \mathbf{s} is the spin vector, and $\mathbf{b} = 3\mathbf{A}/2$. Therefore,

$$\mathbf{b} = \frac{GJ}{r^3} [2 \cos \theta \mathbf{e}_r + \sin \theta \mathbf{e}_\theta] \quad (61)$$

with $J = Ma$ the angular momentum. Denoting $\mathbf{J} = J\mathbf{e}_z$, this equation can be rewritten in the form

$$\mathbf{b} = \frac{G}{r^3} [-\mathbf{J} + 3(\mathbf{J} \cdot \mathbf{e}_r)\mathbf{e}_r]. \quad (62)$$

This means that

$$\mathbf{b} = \boldsymbol{\Omega}_{LT}, \quad (63)$$

where $\boldsymbol{\Omega}_{LT}$ is the Lense–Thirring precession angular velocity, which in general relativity is produced by the gravitomagnetic component of the gravitational field [24]. We see in this way that the axial-vector torsion \mathbf{A} , in teleparallel gravity, represents the gravitomagnetic component of the gravitational field. In fact, considering the slow-rotation and weak-field approximations, the trajectory of a particle, from Eq.(35) with the Kerr torsion, is found to be

$$m \frac{d\mathbf{u}}{dt} = m \left(-\frac{GM}{r^2} \hat{\mathbf{r}} + \mathbf{u} \times \mathbf{A} \right), \quad (64)$$

from where we see clearly that the axial-vector torsion \mathbf{A} is the gravitomagnetic component of the gravitational field.

V. FINAL REMARKS

We have obtained in this paper the teleparallel versions of the Schwarzschild and the stationary axi-symmetric Kerr solutions of general relativity. In the first case, as expected, due to the spherical symmetry of the Schwarzschild solution, the axial-vector torsion vanishes identically. We have then considered the weak-field limit, and we have shown that in this limit the tensor and the vector parts of torsion combine themselves to yield the Newtonian force.

In the second case, we have obtained the torsion tensor, as well as the vector and axial-vector parts of the torsion for the teleparallel Kerr solution. By considering then the slow-rotation and weak-field approximations, we have shown that the axial-vector torsion is nothing but the gravitomagnetic component of the gravitational field, and is therefore the responsible for the Lense–Thirring effect.

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